Electromagnetic radiation from collisions at almost the speed of light: An extremely relativistic charged particle falling into a Schwarzschild black hole

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We investigate the electromagnetic radiation released during the high energy collision of a charged point particle with a four-dimensional Schwarzschild black hole. We show that the spectra is flat, and well described by a classical calculation. We also compare the total electromagnetic and gravitational energies emitted, and find that the former is suppressed in relation to the latter for very high energies. These results could apply to the astrophysical world in the case that charged stars and small charged black holes are out there colliding into large black holes, and to a very high energy collision experiment in a four-dimensional world. In this latter scenario the calculation is to be used for the moments just after black hole formation, when the collision of charged debris with the newly formed black hole is certainly expected. Since the calculation is four dimensional, it does not directly apply to TeV-scale gravity black holes, as these inhabit a world of six to eleven dimensions, although our results should qualitatively hold when extrapolated with some care to higher dimensions.

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I. INTRODUCTION

In a previous paper we have investigated the gravitational radiation emitted by a relativistic particle infalling into a four-dimensional Schwarzschild black hole [1], and afterwards extended it to the infall in a Kerr metric spacetime [2]. The interest of these studies lies in the fact they can describe the gravitational radiation emitted in several collision phenomena, such as the collision between small and massive black holes, between stars and massive black holes, or between cosmic rays and small black holes, to name a few. The possibility of detecting gravitational radiation by the several operating antennae is now real, and a closer understanding of these collisional processes is accordingly important. Of course, to fully understand collisional processes, one has in principle to go beyond the infall of a particle and consider the collision between two black holes with comparable masses, in which case one has to take into account the full nonlinearity and strong field regime of Einstein's equations. Without resorting to numerical computations, but instead using topological arguments, Hawking was able to put upper limits on the gravitational radiation emitted in a black-hole–black-hole collision in four dimensions. This calculation was then refined by D'Eath and Payne [3]. Their calculation made use of the fact that, if one boosts the Schwarzschild metric to high velocities, then it approaches the Aichelburg-Sexl [4] metric, which is a shock wave spacetime describing the gravitational field of a massless particle. It was found that the total efficiency of the shock would be ~16% (see Yoshino and Nambu [5] for the analysis in higher dimensions). In [3], the inclusion of a second term in the news function brings about a decrease in the efficiency from 25% (using only the first term) to 16%, so one is entitled to ask what degree of confidence do these results and techniques offer. For instance, one might suspect the third term to lower even more the efficiency. One needs to have other means of computing this process, since the D'Eath-Payne formalism is hard to pursue, due to its complexity. We have recently [1,2] proposed the ''point particle paradigm,'' which may well be a good candidate. Handling high energy collisions of point particles with black holes is rather simple, since the point particle may be looked at as a perturbation in the black hole spacetime, and the full machinery to handle black hole perturbations is well developed [6]. The interesting thing about the high energy collision of point particles with black holes is that in the limit that the mass μ of the particle goes to the mass M of the black hole, the result for the total radiated gravitational energy agrees well with other predictions and never violates the area theorem [1,2]. Moreover, one can use this method to treat the collision of rotating holes, which does not seem feasible using previous techniques.

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Up to now, most studies have concentrated on the emission of gravitational radiation in the collision process, but if at least one of the particles is charged, electromagnetic radiation is also expected. Astrophysically this is perhaps less relevant for the time being, since the particles one can consider, such as stars or small black holes, are in principle discharged, although it is possible the latter ones can carry some charge (see, e.g., [7]). On the other hand, from the point of view of elementary black holes such a study is certainly of interest: elementary black holes and particles can easily carry electromagnetic charge, and the collision between such objects can be put to operate, either by a machine in some far future that can collide particles at center of mass energies of the order of the four-dimensional Planck mass, 10^{16} TeV, or if there is any fundament in the TeV-scale gravity, by the near future generation accelerators such as the CERN Large Hadron Collider (LHC). The latter case has been given a lot of consideration. The TeV-scale gravity [8] requires extra large dimensions (of the order of submillimeters or smaller) in order to lower the higher-dimensional Planck mass to energies of the order of TeVs. In these TeV-scale gravity models, the gravitons are free to propagate in the higher dimensional spacetime, whereas, due to experimental constraints, the standard model fields live on a 3-brane, our Universe. In this scenario, a particle collider with a center-of-mass energy of the order of TeVs can copiously produce higher dimensional small spherical type black holes, with a radius of the order of fermis or less, in a space with large extra dimensions [9], or even black branes if some of the extra dimensions are large and other small (see [10] for a review). Moreover, in the collision process the gravitons escape to the extra dimensions and are therefore much harder, harder than usual, to detect. On the other hand, all the electromagnetic radiation emitted in such a collision can be detected.

Thus it is important to know the electromagnetic spectrum and the quantity of electromagnetic energy radiated in such an encounter. In this paper we extend the previous calculations into to the electromagnetic window and find the electromagnetic radiation emitted by a highly relativistic electrically charged particle infalling into a four-dimensional Schwarzschild black hole. This is a perturbation calculation: we suppose a small charged particle falling into a Schwarzschild black hole. Some former works that have dealt with the phenomenon of electromagnetic radiation from a charged particle falling from infinity into a Schwarzschild black hole are [11–15]. For example Ruffini [12] first calculated the electromagnetic energy spectra and total radiated energy [12] for particles with low Lorentz factors, $\gamma \ll 1$. Here we go into high γs. We use the point particle approximation and consider a charged point particle colliding head-on at high velocity with a Schwarzschild black hole. We show that the spectra is flat, and we also compare the total electromagnetic and gravitational energies emitted, and find that the former is suppressed in relation to the latter for very high energies. The numerical results extracted by us are in very good agreement with Ruffini's results [12]. We shall also see that there is a classical calculation for this process that agrees extremely well with our numerical results.

Two comments are in order: (i) Our calculation is for the collision of a small particle with a black hole in a four-dimensional world. Therefore, in principle, it could apply to the astrophysical world in case charged stars and charged black holes are out there, and to a very high energy collision experiment in a four-dimensional world (without extra dimensions) and with a fundamental Planck mass of 10^{16} TeV. In this latter scenario the calculation is to be used for the moments just after the black hole formation, when the collision of charged debris with the newly formed black hole is certainly expected. Furthermore, the calculation can be extended to a black hole–black hole collision as we have already argued [1,2], and, in addition, it can give clues, although it does not apply directly to the usual collision process in a collider, i.e., the collision between one non-charged particle with strong gravitational field (not necessarily a black hole) and one charged particle. These results may, however, serve as a model to the electromagnetic radiation emitted in the initial phase of a newly formed black hole, the stage in which the black hole sheds its hair by emitting gauge radiation, such as electromagnetic radiation. (ii) Our calculation is four dimensional. It does not apply to TeV-scale gravity black holes, since these inhabit a world of six to eleven dimensions, with some of the dimensions being large, others perhaps being small. However, although there are certainly some differences, qualitatively our results should hold when extrapolated with some care to higher dimensions.

II. BASIC FORMALISM

In this study we assume the charged particle and the emitted radiation to be a small perturbation on the Schwarzschild spacetime, whose line element is given by

$$ds^2 = -e^{\nu}dt^2 + e^{\lambda}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

where $e^{\nu} = e^{-\lambda} = 1 - 2M/r$, and M stands for the mass of the background spacetime. Because a complete prescription to deal with this problem was given by Zerilli [11], here we briefly show the essential parts of the formalism. By virtue of the static property and spherical symmetry of the background spacetime, perturbations can be decomposed by using vector harmonics for the angular variables θ and φ, and the Fourier components for the time variable t. After accomplishing a separation of variables, perturbations are therefore characterized by the harmonic indices l and m, parity, and frequency ω. For the present case, where a particle falls straight into a black hole along the z axis or the $\theta = 0$ line, no axial parity perturbations are excited due to the symmetry of the motion. Thus, only the polar parity perturbations are considered in this study. According to Zerilli [11], our master equation for determining the electromagnetic radiation is given by a single wave equation,

$$\frac{d^2\tilde{f}_{lm}(\omega,r)}{dr_*^2} + \left[\omega^2 - e^{\nu}\frac{l(l+1)}{r^2}\right]\tilde{f}_{lm}(\omega,r) = e^{\nu}\tilde{S}_{lm}, \tag{2}$$

where r_* is the so-called tortoise coordinate, defined by $dr/dr_* = e^{(\nu-\lambda)/2}$, or $r_* = r + 2M\log(r/2M - 1)$ in our case, and $\widetilde{S}_{lm}$ is the source term determined by the charge and motion of the particle. In the case of the radially falling particle, $\widetilde{S}_{lm}$ is generally given by

$$\widetilde{S}_{lm} = -2q\sqrt{l+\frac{1}{2}}\frac{e^{i\omega T(r)}}{r^2}\delta_{m0}, \tag{3}$$

where δ_{ij} means the Kronecker delta, and q denotes the charge of the particle. Here, the function $T(r)$ is the coordinate time of the particle parametrized by the radial position r of the particle. Since the charged particle is treated as a perturbation of the background spacetime, the particle traces a radial geodesic of the Schwarzschild geometry. Thus, the function $T(r)$ is determined by (see e.g., Chandrasekhar [14])

$$\frac{dT(r)}{dr} = -\frac{e^{(\lambda-\nu)/2}}{\sqrt{1-\gamma^{-2}e^{\nu}}}, \tag{4}$$

where γ is an integral of motion, given by $\gamma=(1-v_\infty^2)^{-1/2}$, where v_∞ is the radial velocity of the particle at spatial infinity.

Once solutions $\widetilde{f}_{lm}(\omega,r)$ of Eq. (2) are calculated, time dependent functions $f_{lm}(t,r)$ can be obtained through the inverse Fourier transformation,

$$f_{lm}(t,r) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-i\omega t}\widetilde{f}_{lm}(\omega,r)d\omega. \tag{5}$$

From the functions $f_{lm}(t,r)$, all the components of the field strength $F_{\alpha\beta}$ can be derived except at the position of the charged particle. For example, the $r-\theta$ component $F_{r\theta}$ is given by

$$F_{r\theta}(t,r,\theta,\varphi) = e^{-\nu}\sum_{l,m} f_{lm}(t,r)\frac{\partial Y_{lm}}{\partial\theta}(\theta,\varphi). \tag{6}$$

In order to obtain a unique solution of Eq. (2), two boundary conditions must be specified, and physically acceptable conditions are a purely outgoing wave at spatial infinity and a purely incoming wave at the horizon, which are due to Eq. (5) given by

$$\widetilde{f}_{lm}(r) \rightarrow \begin{cases} A_{lm}^{\rm in}(\omega)e^{-i\omega r_*} & \text{as} \quad r_* \rightarrow -\infty, \\ A_{lm}^{\rm out}(\omega)e^{i\omega r_*} & \text{as} \quad r_* \rightarrow \infty, \end{cases} \tag{7}$$

where $A_{lm}^{\rm out}(\omega)$ and $A_{lm}^{\rm in}(\omega)$ are not dependent on r. Since we are interested only in solutions at spatial infinity, what we have to do is to obtain $A_{lm}^{\rm out}(\omega)$'s as functions of ω for our purpose of this study. Adapting a standard Green's function technique, we can write $A_{lm}^{\rm out}$ as the integral, given by

$$A_{lm}^{\rm out}(\omega) = \frac{1}{2i\omega C_l(\omega)}\int_{2M}^{\infty} f_{{\rm L},l}\widetilde{S}_{lm}dr, \tag{8}$$

where $f_{{\rm L},l}$ is a homogeneous solution of Eq. (2) satisfying a boundary condition given by

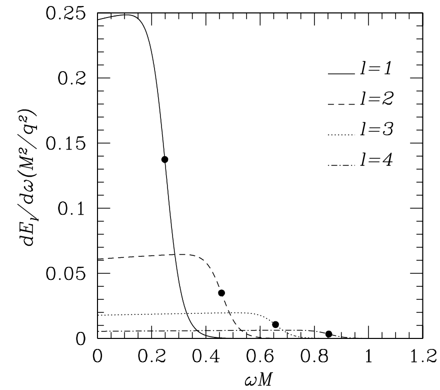

FIG. 1. The electromagnetic energy spectra for the four lowest radiatable multipoles, for a $\gamma=2$ particle falling from infinity into a Schwarzschild black hole. The filled circles on the curves of the energy spectra indicate the frequency of the fundamental quasinormal mode associated with the corresponding harmonic index l.

$$f_{{\rm L},l} \rightarrow \begin{cases} e^{-i\omega r_*} & \text{as} \quad r_* \rightarrow -\infty, \\ B_l(\omega)e^{i\omega r_*} + C_l(\omega)e^{-i\omega r_*} & \text{as} \quad r_* \rightarrow \infty. \end{cases} \tag{9}$$

The energy spectrum at spatial infinity is given by

$$\frac{dE}{d\omega} = \sum_l \frac{dE_l}{d\omega} = \sum_l \frac{l(l+1)}{2\pi}|A_{l0}^{\rm out}(\omega)|^2 \quad \text{for} \quad \omega \geqslant 0. \tag{10}$$

In order to obtain numerical values of $A_{lm}^{\rm out}(\omega)$, we start the integration of $f_{L,l}$ and $\int_{2M}^{r} f_{{\rm L},l}\widetilde{S}_{lm}dr'$ at $r=2M(1+10^{-6})$ by using a Runge-Kutta method. Those functions are then integrated out to large values of r. The integration is stopped if the absolute value of $\int_{2M}^{r} f_{{\rm L},l}\widetilde{S}_{lm}dr'$ converges within the required numerical accuracy, and we simultaneously match $f_{L,l}$ with an asymptotic solution satisfying condition (9) at spatial infinity, which is given in [14], to obtain a value of $C_l(\omega)$.

III. NUMERICAL RESULTS

Following the numerical procedure just outlined, we have computed the energy spectrum for several values of γ. In Fig. 1 we show a typical result of the energy spectrum, here for $\gamma=2$ and for the four lowest radiatable multipoles. In Fig. 2 we show similar results but for a particle with $\gamma \geqslant 10$. A general feature of the energy spectrum for high energy collisions is that it is flat up to some critical frequency, after which it rapidly (exponentially) decreases to zero. This was also verified for the gravitational energy spectrum resulting from the high-energy collision of a black hole with a point particle [1,2]. This critical frequency is given, in a very good approximation, by the frequency of the fundamental quasinormal mode associated with the corresponding harmonic index l. The corresponding quasinormal frequencies are also indicated by a filled circle in Figs. 1 and 2. The total energy ΔE radiated away is given by

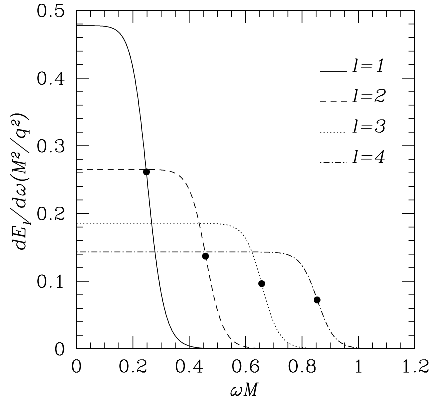

FIG. 2. The electromagnetic energy spectra for the four lowest radiatable multipoles, for a $\gamma \gg 10$ particle falling from infinity into a Schwarzschild black hole. The filled circles on the curves of the energy spectra indicate the frequency of the fundamental quasinormal mode associated with the corresponding harmonic index l.

$$\Delta E = \sum_{l=1}^{\infty} \Delta E_l , \tag{11}$$

where ΔE_l is the total radiated energy associated with l-pole electromagnetic radiation, which is given by the area under the curve of the energy spectrum $dE_l/d\omega$ (see Fig. 1). For the low γ cases, the convergence of the infinite series (11) is relatively good in the sense that a finite summation of several ΔE_l's is sufficient to obtain a value of ΔE with sufficient accuracy. However, the convergence of the total radiated energy gets worse with increasing γ. This contrasts with the analogous calculation of the gravitational energy radiated in high energy collisions [2] where a rescaling was possible to show that $\Delta E \propto \gamma^2$. Here we shall see in the next section that the difficulty for the electromagnetic case lies in the fact that $\Delta E \propto \log \gamma$, for high γ's. For a very fast particle, $\gamma \sim \infty$, it is quite difficult to numerically obtain the total radiated energy, since all the l-pole energy spectra $dE_l/d\omega$ make a substantial contribution to the total radiated energy. In this study, we calculated the total energy for several values of γ with $\gamma \leqslant 10$. Some of these values are shown in Table I.

Previous works concerned with the electromagnetic radiation from a charged particle falling from infinity into a Schwarzschild black hole include for example Ruffini [12]. Ruffini first calculated the electromagnetic energy spectra and total radiated energy [12], for low γ particles. The numerical results extracted by us are in very good agreement with Ruffini's results [12].

TABLE I. The total radiated electromagnetic energy as a function of γ.

γ	$\Delta E M q^{-2}$
1	2.14×10^{-2}
2	1.14×10^{-1}
3	2.23×10^{-1}
4	3.34×10^{-1}
5	4.46×10^{-1}

IV. THE CLASSICAL CALCULATION

There is a classical calculation in electromagnetism that suggests itself as a model to use in the high energy collision of charged particles: the radiation emitted when a charge, with constant velocity v_∞, is suddenly decelerated to $v=0$. The deceleration is idealized as taking zero seconds. This model calculation has been applied with great success to, for example, beta decay [16,17]. The result for the energy spectrum per solid angle is [16,17]

$$\left(\frac{d^2E}{d\omega d\Omega}\right)_{\rm class} = \frac{q^2 v_\infty^2}{4\pi^2} \frac{\sin^2\theta}{(1-v_\infty \cos\theta)^2}, \tag{12}$$

or, integrating over solid angle,

$$\left(\frac{dE}{d\omega}\right)_{\rm class} = \frac{q^2}{\pi}\left[\frac{1}{v_\infty}\log\left(\frac{1+v_\infty}{1-v_\infty}\right) - 2\right]. \tag{13}$$

This formula has been tested with great accuracy experimentally. To get the total energy, one has to integrate (13) over frequencies, and a naive procedure would lead to infinities in the total energy. To obtain reasonable results, one has to impose a cutoff ω_c in the frequency depending on the particular problem under consideration, and one obtains

$$\Delta E_{\rm class} = \frac{q^2}{\pi}\left[\frac{1}{v_\infty}\log\left(\frac{1+v_\infty}{1-v_\infty}\right) - 2\right]\omega_c . \tag{14}$$

How well do these classical formulas fit into our numerical results? Very well indeed. To allow a more direct comparison, we shall first decompose the energy spectrum (13) into spherical harmonics, i.e., into the energy spectrum associated with each l-pole radiation as follows:

$$\left(\frac{dE}{d\omega}\right)_{\rm class} = \sum_l \left(\frac{dE_l}{d\omega}\right)_{\rm class} = \sum_l \frac{q^2}{4\pi^2 l(l+1)}\left|\int \frac{v_\infty \sin\theta}{1-v_\infty\cos\theta}\frac{\partial Y_{l0}}{\partial\theta} d\Omega\right|^2 . \tag{15}$$

When we consider the limit of $v_\infty \to 1$, as shown in Eq. (13), the total energy spectrum $(dE/d\omega)_{\rm class}$ diverges. However, the energy spectrum due to l-pole radiation $(dE_l/d\omega)_{\rm class}$ converges to a finite value, given by

$$\left(\frac{dE_l}{d\omega}\right)_{\rm class} = q^2 \frac{2l+1}{\pi l(l+1)} . \tag{16}$$

This yields, for example, $dE/d\omega_{l=1} = 0.477q^2$ and $dE/d\omega_{l=2} = 0.265q^2$ as $\gamma \to \infty$. This may be compared with the numerical values in Fig. 2. The numerical agreement is excellent not only for these lower multipoles, but also for higher ones, and one has therefore established numerically that for high energy collisions the classical result (13) for the energy spectrum is valid. Probably more interesting is the total energy radiated. Since one already knows that the clas-

TABLE II. The cutoff frequency ω_c as a function of γ.

γ	$\omega_c M$
1	∞
2	3.44×10^{-1}
3	4.03×10^{-1}
4	4.64×10^{-1}
5	5.23×10^{-1}

sical result for $dE/d\omega$ is correct, let us now try to predict the cutoff frequency by identifying ΔE in Eq. (11) with Eq. (14). Making use of this definition of ω_c and numerical values of ΔE, we evaluate the cutoff frequencies ω_c in Eq. (14) for several values of γ, and list the values of ω_c in Table II. In Fig. 3 we also show the cutoff frequencies as a function of γ. It is found in this figure that for relatively high values of γ, ω_c increase almost linearly with γ, and a good fit to our numerical values is given by

$$M\omega_c = 0.224 + 0.0598\gamma. \tag{17}$$

In Fig. 3 we have shown this linear function, and we can confirm that this linear function is in good agreement with our numerical values for high γ. This means that a good approximation to the total electromagnetic energy radiated is

$$\Delta E = \frac{q^2}{M\pi}\left[\frac{1}{v_\infty}\log\left(\frac{1+v_\infty}{1-v_\infty}\right)-2\right](0.224+0.0598\gamma), \tag{18}$$

or considering that $v_\infty \sim 1$ we have also

$$\Delta E = \frac{2q^2}{M\pi}[\log 2\gamma - 1](0.224+0.0598\gamma), \tag{19}$$

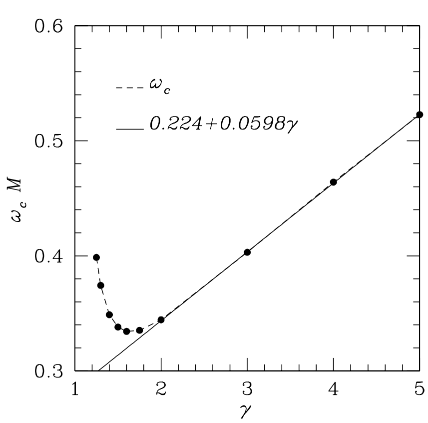

FIG. 3. The cutoff frequency, given as a function of γ. The filled circles on the curve of the cutoff frequency indicate the cutoff frequency calculated in this study. The linear function $\omega_c = 0.224 + 0.0598\gamma$ is also shown.

V. COMPARING THE AMOUNT OF GRAVITATIONAL AND ELECTROMAGNETIC ENERGY RADIATED

Previous studies on the high energy collision of point particles with black holes led [1,2] to the following conclusion: if a high energy point particle of mass μ collides head on with a mass M non-rotating black hole, the total amount of gravitational energy radiated is

$$\Delta E_g = 0.26\frac{\mu^2\gamma^2}{M}, \tag{20}$$

where γ is again the Lorentz factor for the point particle. Furthermore, it was also found that the spectra is flat and also well described by a classical calculation (here classical means again working on a flat background). Now, for high γ's (19) is

$$\Delta E \sim \frac{0.038q^2}{M}\gamma\log 2\gamma, \gamma\to\infty. \tag{21}$$

So, we get for the ratio electromagnetic energy over gravitational energy,

$$\frac{\Delta E}{\Delta E_g} \sim 0.146\left(\frac{q}{\mu}\right)^2\frac{\log 2\gamma}{\gamma}, \gamma\to\infty. \tag{22}$$

This means that electromagnetic energy is suppressed in relation to gravitational energy, for very high energies.

VI. CONCLUSIONS

We have computed the electromagnetic spectrum and total energy radiated during the high energy collision of a charged point particle with a Schwarzschild black hole. Our results show that the classical ‘‘instantaneous collision’’ calculation gives very good results, in accordance with the full numerical ones. We have dealt only with zero impact (i.e., head-on) collisions, but these results are easily generalized to non-head-on collisions. For example, one expects that the classical results [16,17] hold for such cases, and therefore that the total energy decreases as one increases the impact parameter. We stress that the results presented here are valid for any particle charge q, as long as the total effective stress energy of the particle is small compared to the total energy content of the black hole. Previous works [1,2] have shown that the high energy collision between two black holes of equal mass may be well studied through the collision of a point particle with a black hole, and then taking the limit of equal mass, although this is formally not allowed. It is our belief that the same may be done here. One needs, however, some other method to attack this problem, to confirm or disprove this claim. The investigation carried here can also be carried over to higher dimensions, a case which is of more direct interest to TeV-scale gravity scenarios. Since standard model fields inhabit a four-dimensional brane, what one would need to generalize this construction would be to modify the induced four-dimensional metric (1), as was done, for example, in [18]. The generalization is straightforward.

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